

# Designing Cognitively Demanding Problems

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# Learning Outcomes (Goals)

- You will identify and be able to apply criteria to evaluate the level of cognitive demand of math problems
- You will learn, practice and be able to use specific techniques to increase the level of cognitive demand of the problems you assign to your students

# What We Will Do Today:

- **Activity 1: Sorting Problems by Cognitive Demand → Identifying Criteria**
- **Activity 2: Designing Cognitively Demanding Problems**
- **Activity 3: Looking at Mathematical Processes During Problem Solving (Connections to the CCSS-M)**

# Activity 1: Sorting Problems by Cognitive Demand

- Solve problems with higher cognitive demand.
- Determine features that distinguish problems with higher cognitive demand within each group (A, B, C, and D).
- Discuss these features with your partner(s).
- Share your group's thoughts.

# Activity 2: Designing Cognitively Demanding Problems

- **Technique 1:** Design a mathematical investigation to discover an important formula or relationship.
- **Example: #3a-e.** Creating and observing a pattern, formulating a hypothesis, justifying, generalizing, and proving.

# Activity 2: Designing Cognitively Demanding Problems

- **Technique 2:** Combine problems that develop and build one idea and have students analyze, compare and contrast these problems together.
- **Example:** 2. Consider the linear functions represented graphically on the next page.
  - a. Sort these functions into two or three groups based on common features among them.
  - b. Explain the criteria for your sorting. What do the functions in each group have in common? What are some differences between functions within the same group?

# Activity 2: Designing Cognitively Demanding Problems

- **Technique 3:** Invert a problem: The answer becomes “what is given” in a problem, and “the given” becomes what has to be found or solved for.
- **Example:** Write at least three proportions that have the cross-product

$$6 \cdot 2x = 5a \cdot 7$$

# Activity 2: Designing Cognitively Demanding Problems

- **Technique 4:** Increase the number of “steps” required to reach the solution without specifying the sequence of these steps. Require explanations/justification for the solution plan.
- **Example:** #9 requires 6 steps (repeated twice) + 7<sup>th</sup> step – comparison – to solve, and the path for solution is not given.



# Activity 2: Designing Cognitively Demanding Problems

- Apply the techniques specified above to modify at least two problems below or another problem of your choice in order to create problems with a higher level of cognitive demand.
- Write your modified problems on a poster and put it on the wall.

# CCSS Mathematical Practice #1:

*Make sense of problems and persevere in solving them.*

1. Explain the meaning of a problem and look for entry points to its solution
2. Analyze givens, constraints, relationships, and goals of the problem
3. Consider analogous problems, and try special cases and simpler forms of the original problem
4. Monitor and evaluate progress (in solving the problem) and change course if necessary

# CCSS Mathematical Practice #1 (continued)

5. Explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.
6. Rely on using concrete objects or pictures to help conceptualize and solve a problem.
7. Check the answer(s) to a problem using a different method, and continually ask, “Does this make sense?”
8. Understand other peoples’ approaches to solving complex problems and identify correspondences between different approaches.

# Activity 3: Looking at Mathematical Processes During Problem Solving

- Discuss with your group:
- What are the two or three problems which use the highest numbers of the processes listed in the table below?
- Are there any processes in the table of Mathematical Processes (that you have not yet listed) that are or could be at work in problems that you classified into the higher cognitive demand category?

# Activity 3: Using Questioning

- Insert additional questions into the following problem to increase its cognitive demand:

# Activity 4: Homework

- Evaluate the problems you will give to your students in the next week or two to determine their level of cognitive demand.
- Specify which of the mathematical processes these problems use and which processes are lacking.
- Modify some of the problems to increase their cognitive demand using tools from today's session.

# And Beyond...

- Step 1 = Select or design cognitively demanding problems
- What happens when implemented?
- See handout for strategies to keep in mind to maintain high levels of cognitive demand while implementing problems.

# High Leverage Instructional Practices Linked to CCSS Mathematical Practices

## Instruction that:

- approaches mathematics learning as problem solving (MP 1)
- **emphasizes cognitively demanding conceptual tasks that encourage all students to remain engaged in the task without watering down the expectation level (maintaining cognitive demand) (MP 1)**
- places the highest value on student understanding (MP 1 and 2)
- emphasizes the discussion of alternative strategies (MP 3)
- includes extensive mathematics discussion (math talk) generated through effective teacher questioning (MP 2, 3, 6, 7, and 8)
- elicits student explanations to support strategies and conjectures (MP 2 and 3)
- emphasizes the use of multiple representations (MP 4 and 5)



# Thank you!

## For more information:

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